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A theoretical essay for the enlargement of the knowledge

on the optimal state of a fishery.

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All criticisms and suggestions shall be welcome!

Introduction

All along the current literature on this task, some theorems are available for the determination of the optimal biomass of a fishery. However, optimal biomass is defined in such theorems as the amount of biomass that maximizes the present value of future profits, though quantification and subsequent verification of such maximum is not possible.

In addition, in such literature some deficiencies are observed: Generally, the price of the resource is supposed to be given but, actually, there exists a growing demand, which will be increased in the future. Frequently, the presence of a fixed cost in the fishing activity is neglected. In the present essay, we will attempt to give solutions for both problems.

New theoretical essay

For our purposes, we will start from the traditional planning, in this sort of studies1, which will added a demand function, $p = p(h),\ \ dh/dp < 0$ and the fixed cost FC. As usual, under dynamic formulation, optimal biomass x^* is defined as the value of x for which:

$$PV = \int_0^\infty e^{-\delta t}\left[p(h)h - \left(FC + c(x)h\right)\right]dt \quad (1) \quad \text{with condition given by: } \frac{dx}{dt} = F(x) - h$$

is a maximum.

As, in the present, it is generally known that fisheries are subject to an excess of effort, that is to say $x_0 < x^*$, in this essay it will be supposed that, in order to reach the optimal biomass x^*, from initial biomass x_0, the fishing authority will demand the fastest economic policy, that is:

Transient stage. From $t = 0$ to $t = \tau$ a prohibition of catches will be imposed ($h = 0$), so that, in this stage, biomass will follow its natural growth: $dx/dt = F(x)$.

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[1] This planning is based, fundamentally, in the so-called “Gordon and Schafer traditional model”.

Permanent stage. When $t > \tau$, a TAC equal to natural annual growth will be imposed ($h^* = F(x^*)$), and biomass will remain in the optimal stationary state: $dx / dt = 0$.

When following this policy, the previous expression (1) can be divided into both cited stages:

$$PV = \int_0^{\tau} e^{-\delta t}(-FC)dt + \int_{\tau}^{\infty} e^{-\delta t}\left[p^* h^* - \left(FC + c(x^*)h^*\right)\right]dt$$

Integrating this expression we will get:

$$PV = \frac{-FC}{\delta} + \frac{h^* e^{-\delta\tau}\left(p^* - c(x^*)\right)}{\delta} \quad (2)$$

Moreover, having in mind that natural growth of biomass is: $\frac{dx}{dt} = F(x)$, the duration τ of the transient regulation stage will be:

$$\tau = \int_{x_0}^{x^*} \frac{1}{F(x^*)} dx^* \quad (3)$$

Therefore, substituting $h^* = F(x^*)$ and expression (3) in (2), we get a relation in which:

$$PV = PV(x^*, p^*) \quad (4)$$

In order to maximize this expression, as demand has a finite elasticity, regulation could pursue one of the following objectives:

Objective I.- Maximization of the total surplus of the fishing activity (producers' profit plus consumers' surplus).

In this case, which is the most frequent, regulation will attempt to get a similar optimal result comparable to the maximum in a competitive market among all fishing enterprises.

Consequently, in order to get the maximum, we will get the derivative of (4):

$$\frac{\partial PV}{\partial x^*} = PV_x^{'}(x^*, p^*)$$

Next, we will make $p^* = p(h^*)$, where $h^* = F(x^*)$. Therefore, price is fixed by the equality between demand and supply, and none of the fishing enterprises would have influence on it. After, this expression is equal to zero. Solving δ in it, we will get the following relation:

$$\delta = F'(x^*) - \frac{c'(x^*)F(x^*)}{p(h^*) - c(x^*)}, \text{ where } h^* = F(x^*) \quad (5)$$

This relation, given the discount rate δ, allows obtaining the optimal biomass for Objective I. Using this value, the rest of the endogenous variables of this model can be determined.

Objective II.- Maximization of the producers' profit.
In this case, regulation would try to get a similar result analogous to the maximum in the sole-owner situation, that is, all fishing enterprises belong to the same monopolist company.

When attempting to reach this, $p^* = p(h^*)$, with $h^* = F(x^*)$ will have to be substituted in (4), provided that the monopolist has the ability to fix the price. Hence, optimal price will maximize the monopolist's profit.

Next, we will get the derivative of the obtained expression and finally, we will equal it to zero. Solving for δ, we will get this relation:

$$\delta = F'(x^*) - \frac{[c'(x^*) - F'(x^*)p'(h^*)]F(x^*)}{p(h^*) - c(x^*)}, \text{ where } h^* = F(x^*). \quad (6)$$

Given the discount rate, this relation allows us getting the optimal biomass for Objective II. After, the rest of the endogenous variables can be found.

Advantages of this new approach

This essay makes evident that expressions (5) and (6), depending on the desired objective, permits us to determine the optimal biomass. But, also:

expression (3) yields the length of the transient stage.

expression (2) facilitates the value of maximum PV and allows proving that it is a maximum.

These argumentation let us say that this new way can grant some advantages to Fisheries Research, such as:

Possibility of evaluation of different economic policies.

A transient regulation stage which implies the prohibition of catches during a certain period of time, obviously, must be accompanied of other measures such as financial assistance to ship owners (at least, the fixed costs) and fishermen doles. This new way can evaluate such measures.

Of course, some other alternative policies can be proposed instead of a total prohibition of catches during the transient regulation stage (which will be less traumatic for all social sectors), such as fixed TACs, TACs equal to a percentage of biomass natural growth, etc. This new way, as it includes financial assistance, can evaluate each policy and compare it with other alternatives.

Possibility of detecting non-voluntary errors

When we began this researching line, we noticed that, along the current literature, optimal biomass is considered to be the quantity of biomass that maximizes PV, according to a certain theorem. However, this theorem cannot quantify and prove such maximum PV.

Clark[2], after other authors, gives the following solution to this theorem.

Supposing price to be constant, and using the maximum principle of Pontryagin[3], he deduces this expression for the optimal biomass:

$$\delta = F'(x^*) - \frac{c'(x^*)F(x^*)}{p - c(x^*)}$$

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[2] Clark, C. W. (1990) *Mathematical Bioeconomics. The Optimal Management of Renewable Resources.* New York. John Wiley & Sons.

[3] Pontryagin L.S. et al. (1962). *The Mathematical Theory of Optimal Processes.* New York. Wiley-Interscience

Solving for price, he gets a discounted supply curve:

$$p = c(x) - \frac{c'(x^*)F(x^*)}{\delta - F'(x^*)} = H_\delta(x^*) \text{, where } x^* = F^{-1}(h^*) \qquad (7)$$

Supposing price to be variable, according to a demand schedule $p = p(h)$, analyzes both possible cases.

Competitive case (our Objective I).

In this case, Clark equals demand and discounted supply, obtaining:

$$p(h^*) = c(x^*) - \frac{c'(x^*)F(x^*)}{\delta - F'(x^*)} \text{, where } x^* = F^{-1}(h^*)$$

That is to say:

$$\delta = F'(x^*) - \frac{c'(x^*)F(x^*)}{p(h^*) - c(x^*)} \text{, where } h^* = F(x^*)$$

This relation, as it can be observed, is identical to ours (5). On submitting this expression to the verification proof, here exposed, we can see that the value of biomass x^* gives the maximum PV, and so, it is the optimal biomass.

Monopoly case (our Objective II).

In this case, Clark equals marginal revenue and discounted supply, so that:

$$\frac{d\left(p(h^*)h^*\right)}{dh^*} = c(x^*) - \frac{c'(x^*)F(x^*)}{\delta - F'(x^*)} \text{, where } x^* = F^{-1}(h^*) \qquad (8)$$

Solving this expression, he gets:

$$\delta = F'(x^*) - \frac{c'(x^*)F(x^*)}{\left[F(x^*)p'(h^*) + p(h^*)\right] - c(x^*)} \text{, where } h^* = F(x^*)$$

We can already see that this expression, which attempts to get the same objective as ours (6), does not coincide with that. On submitting both expressions to this verification proof, the results are the following:

Using our expression (6), the yielded biomass corresponds to the maximum PV, so we can say that it is the optimal biomass.

Using Clark's expression, the obtained biomass gives a lower value of PV, so it is not the optimal biomass.

Yet, another advantage of this complementary way is the possibility of detecting non-voluntary mistakes.

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